Optimization Theory and Algorithm	**Homework 2 - 05/07/2021**

Homework 2

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HW 1 *In the Portfolio Management example, we formulate the oprimization problem as:*

$$\max_{\mathbf{x}} \ \mathbb{E}(R) - \lambda Var(R), \tag{1}$$

$$s.t. \ x_i \geq 0, i = 1 \ldots, n, \tag{2}$$

$$\sum_{i=1}^{n} x_i = 1, \tag{3}$$

where $\mathbb{E}(R)$ is the expectation of R, $Var(R)$ is the variance of R and $\lambda > 0$ is called risk aversion parameter for balancing the investment risk and expected return. We further suppose that $\mathbf{r} = (r_1, \ldots, r_n)^\top$ is a random vector with expectation of $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_n)^\top$, and covariance of Σ. Then prove that the optimization problem (1) is equivalent to

$$\max_{\mathbf{x}} \ \boldsymbol{\mu}^\top \mathbf{x} - \lambda \mathbf{x}^\top \Sigma \mathbf{x}, \tag{4}$$

$$s.t. \ x_i \geq 0, i = 1 \ldots, n, \tag{5}$$

$$\sum_{i=1}^{n} x_i = 1. \tag{6}$$

HW 2 *Poisson Regression: Suppose now that the b_i are counts and the $\mathbf{a}_i$ are some predictors in $\mathbb{R}^p$. Let λ_i be the expected number of events in sometime period, i.e.*

$$P(b_i|\lambda_i) = \frac{e^{-\lambda_i}\lambda_i^{b_i}}{b_i!}.$$

Based on the example of Generalized Linear Model, please derive the following optimization form to present the simple Poisson regression.

$$\min_{\mathbf{x}} \ \sum_{i=1}^{m} \exp(\langle \mathbf{a}_i, \mathbf{x} \rangle) - \langle \mathbf{b}, A\mathbf{x} \rangle.$$

HW 3 *Let*

$$f(\mathbf{x}) = \sum_{i=1}^{m} \log(1 + \exp(\langle \mathbf{a}_i, \mathbf{x} \rangle)) - \langle \mathbf{b}, A\mathbf{x} \rangle$$

be the objective function of logistic regression, please compute the gradient and Hessian matrix of $f(\mathbf{x})$.

HW 4 *The textbook, Page 22, 1.4.*

HW 5 *Please implement Jacobi and Gauss-Seidel iterative algorithms for solving the following equations:*

$$A\mathbf{x} = \mathbf{b},$$

where

(1) $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{pmatrix}$ *and* $\mathbf{b} = (2, 12, 2)^\top$.

(2) $A = \begin{pmatrix} 8 & -3 & 2 \\ 4 & 11 & -1 \\ 2 & 1 & 4 \end{pmatrix}$ *and* $\mathbf{b} = (20, 33, 12)^\top$.

Please discuss the convergence conditions of these two equations.

References