Optimization Theory and Algorithm II **September 22, 2022**

Homework 2

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HW 1 *Consider the transportation problem:*

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}c_{ij}, \tag{1}$$

$$s.t.\ x_{ij} \geq 0, i = 1, \ldots, m; j = 1, \ldots, n, \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \tag{3}$$

$$\sum_{i=1}^{m} x_{ij} = b_j. \tag{4}$$

Show its standard form of LP.

HW 2 *Consider a linear programming*

$$\begin{aligned} &\min\ \mathbf{c}^\top \mathbf{x}, \\ &s.t.\ A\mathbf{x} = \mathbf{b}, \\ &\quad \mathbf{x} \succeq 0. \end{aligned}$$

(i) Show its Lagrange dual problem and KKT conditions.

(ii) Using its KKT conditions to show that the strong duality holds.

HW 3 *Let us consider*

$$\begin{aligned} \min_{\mathbf{x}}\ &\mathbf{c}^\top \mathbf{x} - \mu \sum_i \log x_i, \\ s.t.\ &A\mathbf{x} = \mathbf{b}. \end{aligned}$$

(i) Show its KKT conditions.

(ii) Show the explicit form of secant equation of the KKT system.

HW 4 *Consider the following linear programming*

$$\begin{aligned} \min\ &-5x_1 - x_2 \\ s.t.\ &x_1 + x_2 \leq 5, \\ &2x_1 + x_2/2 \leq 8, \\ &x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

(i) Add slack variables x_3 and x_4 to convert this problem to standard form.

(ii) Implement the simplex method to solve this problem.

(iii) Implement the interior point method to solve this problem.

HW 5 *Implement the interior point method to solve*

$$\begin{aligned} &\min\ x_1^2 + 2x_2^2 - 2x_1 - 6x_2 - 2x_1x_2 \\ &s.t.\ x_1/2 + x_2/x \le 1,\ -x_1 + 2x_2 \le 2, x_1 \ge 0, x_2 \ge 0. \end{aligned}$$

References