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| **Optimization Theory and Algorithm** | **October 7, 2021** |

# Homework 2

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**HW 1** *Consider a linear programming*

$$\begin{aligned} &\min\ \mathbf{c}^\top \mathbf{x}, \\ &s.t.\ A\mathbf{x} = \mathbf{b}, \\ &\quad \mathbf{x} \succeq 0. \end{aligned}$$

*(i) Show its Lagrange dual problem.*

*(ii) Using its KKT conditions to show that the strong duality holds.*

**HW 2** *Consider the following linear programming*

$$\begin{aligned} &\min\ -5x_1 - x_2 \\ &s.t.\ x_1 + x_2 \leq 5, \\ &\quad 2x_1 + x_2/2 \leq 8, \\ &\quad x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

*(i) Add slack variables $x_3$ and $x_4$ to convert this problem to standard form.*

*(ii) Implement the simplex method to solve this problem.*

*(iii) Implement the interior point method to solve this problem.*

**HW 3** *First, recall the support vector machine in the lecture notes. Then consider the following relaxed linear SVM model*

$$\begin{aligned} &\min_{\mathbf{w},\mathbf{b},\xi} \frac{\|\mathbf{w}\|^2}{2} + C\sum_{i=1}^n \xi_i \\ &s.t.\ y_i(\langle \mathbf{w}, \mathbf{x}_i \rangle + \mathbf{b}) \geq 1 - \xi_i, \\ &\quad \xi_i \geq 0, i = 1, \ldots, n, \end{aligned}$$

*where $C$ is a constant.*

*Please show its Lagrange dual problem.*

**HW 4** *Implement the interior point method to solve*

$$\begin{aligned} &\min\ x_1^2 + 2x_2^2 - 2x_1 - 6x_2 - 2x_1x_2 \\ &s.t.\ x_1/2 + x_2/x \leq 1,\ -x_1 + 2x_2 \leq 2, x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

**HW 5** *The problem of finding the shortest distance from a point $\mathbf{x}_0$ to the hyperplane $\{\mathbf{x}|A\mathbf{x} = \mathbf{b}\}$, where $A$ has full row rank, can be formulated as the quadratic program*

$$\begin{aligned} &\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{x}_0 - \mathbf{x}\|^2 \\ &s.t.\ A\mathbf{x} = \mathbf{b}. \end{aligned}$$

*(i) Show that the optimal Lagrange multiplier is*

$$\boldsymbol{\nu}^* = (AA^\top)^{-1}(A\mathbf{x}_0 - \mathbf{b}).$$

*(ii) Show that optimal solution is*

$$\mathbf{x}^* = \mathbf{x}_0 + A^\top(AA^\top)^{-1}(A\mathbf{x}_0 - \mathbf{b}).$$

# References