Optimization Theory and Algorithm	**Lecture 15 - 06/15/2021**

Homework 1

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HW 1 *Consider the following convex optimization problem:*

$$\min_{\mathbf{x}} f(\mathbf{x}),$$
$$s.t.\ \mathbf{x} \preceq 0.$$

Then, according to general optimality condition prove that the optimal point $\mathbf{x}^*$ *satisfies*

$$\nabla f(\mathbf{x}^*) \preceq 0 \text{ and } x_i^*(\nabla f(\mathbf{x}^*))_i = 0, i = 1, \ldots, n.$$

HW 2 *Consider a linear programming*

$$\min \mathbf{c}^\top \mathbf{x},$$
$$s.t.\ A\mathbf{x} \leq \mathbf{b}.$$

Show its Lagrange dual problem.

HW 3 *Compute the conjugate funtion:*

(i) $f(\mathbf{x}) = \delta_{B_\infty}(\mathbf{x})$.

(ii) $f(\mathbf{x}) = \delta_{\mathbb{R}^n_-}(\mathbf{x})$.

(iii) $f(x) = \log(1 + \exp(x))$.

(iv) $f(\mathbf{x}) = g(\mathbf{x} - \mathbf{a}) + < \mathbf{x}, \mathbf{b} >$.

(vi) $f(\mathbf{x}) = \inf_{\mathbf{z}}\{\frac{1}{2}\|\mathbf{x} - \mathbf{z}\|^2 + g(\mathbf{z})\}$.

HW 4 *Consider the following problems:*

(i) Define the negative entropy function is $f(x) = x \log x$ *and* $x \geq 0, 0 \log 0 = 0$*, compute its conjugate function.*

(ii) Consider the following entropy maximization problem:

$$\min_{\mathbf{x}} \sum_i x_i \log(x_i),$$
$$s.t.\ A\mathbf{x} \succeq \mathbf{b},$$
$$\sum_i x_i = 1.$$

Please compute its Lagrange dual problem.

(iii) Suppose the strong duality holds for the entropy maximization problem, and we have obtained the optimal dual variables $\boldsymbol{\lambda}^*$ *and* $\boldsymbol{\nu}^*$*, then compute the optimal primal variable* $\mathbf{x}^*$ *by* $\boldsymbol{\lambda}^*$ *and* $\boldsymbol{\nu}^*$*.*

HW 5 *Show the Lagrange dual problems for*

(i)

$$\min_{\mathbf{x}} f(\mathbf{x}) + g(A\mathbf{x}).$$

(ii) Ridge Regression:

$$\min_{\mathbf{x}} \frac{1}{2}\|A\mathbf{x} - \mathbf{b}\|^2 + \lambda\|\mathbf{x}\|_2^2.$$

References